NONLINEAR METHOD OF DETERMINING THE VEHICLE PRE-CRASH SPEED BASED ON B-SPLINE WITH PROPABILISTIC WEIGHTS – SUBCOMPACT CAR CLASS

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Abstract

A new non-linear method utilizing the work W of car deformation is considered in this study. The car deformation is defined as an algebraic function of deformation ratio Cs.
The method of variable correlation is exploited in order to develop experimental data. To determinate mathematical model parameters, data from the NHTSA database including frontal crash tests are used. In the non-linear method used so far, the so-called energetic approach, collisions are considered non-elastic.
The speed threshold defining the elastic collision was set to be 11 km/h. This simplistic approach is used to determine the linear relation of energy loss during deformation on deformation coefficient Cs. Deformation points C1-C6 are taken into account while calculating a mean value that defines this coefficient. A more accurate non-linear method as well as more complex form of deformation coefficient is suggested to determine the work of deformation in this paper.

Keywords: air braking systems dynamic, agricultural trailer, braking system optimization, air brake valve

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Nomenclature:

EES – Equivalent Energy Speed [m/s]
NHTSA – National Highway Traffic Safety Administration
C_s – deformation ratio [m]
C_1-C_6 – deformation coefficients
E/M – specific collision energy [J/kg]
L_t – dent zone width [m]
V_t – vehicle speed [m/s]
W_{def} – work of deformation [J]
b_k – constant slope factor [m/s/m]
b_{sg} – limit speed [m/s]
m – weight of car [kg]
n – number of cases [-]
error – EES estimation error [-]
A [N/m], B [N/m2], G [N], C [m], α [m], β [m^2] – coefficient

1. Introduction

The presented paper studies the SubCompact Car Class using a new non-linear method. This method is based on the work W of vehicle deformation which is determined by the algebraic function of the generalized deformation Cs of the car body [20,22,36]. The basis of the proposed method is the approximation with B-spline tensor products. What is more, probabilistic weights in the least-squares method are introduced.

The determination of pre-crash velocity V_r=EES [6,7,37] of the tested vehicle is needed for each crash reconstruction. Not only is it necessary to perform a time-space analysis of the crash course, but also an assessment of its mitigation and evaluation of the tactic and technique of the driver is required [2,21,27]. There are a few ways to determine the EES velocity [5,30,34] of vehicles that takes part in a collision. In each case, the preferred method is chosen depending on the type of collision and data collected. Three fundamental models of car accidents can be differentiated:

- Pedestrian hit by a vehicle
- Single track vehicle accidents
- Vehicle collision.

When a vehicle crash into an obstacle, some kind of chassis deformation always occurs [30,31,35]. The scale of deformation is strictly connected with the work that is necessary for its accomplishment. When a car strikes an immovable obstacle, deformation work is the kinetic energy that is lost during the collision [16,24]. When the work used for deformation is known, it is possible to determine the vehicle speed before the impact according to the following relation (used only if the vehicle stopped upon hitting the obstacle).

Once the above parameters are established, the deformation work can be defined. This quantity is then directly used to find the value of EES from the following equation:

$$V_r = \sqrt{\frac{2 * W}{m}}, \tag{1}$$

where: V_r – vehicle velocity before impact, W – deformation work, m – vehicle weight.

If calculation of the deformation work W is possible, the pre-crash speed can be easily determined. Obtaining the proper data for each car brand may cause some difficulties, mainly because crash tests are not always published or, if published, data processing is really complex. Energy absorption structure specifics determined from crash tests has to be known to determine precisely the deformation work of each car. They present crash data for the front, back and side of a vehicle, divided into segments e.g. C1-C6. The data acquired for each point in a given class of the car cannot be used to determine the energy absorption in other vehicle classes or mass ranges. For every test, the most important thing is to photograph and measure the deformation of the vehicle precisely [13-15]. The estimation of deformation work for each vehicle is possible using the method developed by Walter Röhrich.

Repeated crash tests with the assumption of frontal vehicle collision are included in the NHTSA database [28,29]. The linear method is an energy one and it assumes that such collisions are inelastic. The collision can be considered elastic when the threshold is assumed to be determined by a speed of 11 km/h. Such simplified approach characterize the linear relation between the energy loss during deformation that occurs as a result of a car crash and the generalized deformation coefficient Cs [4,8,23]. This coefficient is defined as a mean value of smaller weights for deformation points C1-C6 obtained while measuring the deformation after the car crash. The points C1-C6 describe changes of a car body profile within the dent zone width [9,17,25].

A more accurate non-linear method that enables the determination of deformation work during a crash as well as a more developed form of determining the deformation coefficient is suggested in this paper.

The rule in such an approach is not only to determine the function of deformation work W against deformation coefficient Cs, but also to determine the relation between the deformation coefficients C1-C6 for each crash case [26,32,33]. As a result, it is shown that the relation between deformation work W and deformation coefficient Cs is non-linear. As for the SubCompact Car Class, deformation work W decreases with propagating deformation. As a result, the relation can be described by a hyperbolic curve.

This is connected with the fact that the vehicle loses its stiffness successively during the crash as the following elements of the front part of the car body are broken and the cab itself is further deformed. Lower work W is needed to overcome the same deformation path. As a result, a relation between the work W and coefficient Cs exists [10-12].

In the entire range of Cs coefficients there is a strong and evident relation. During research, such an approach being used as plots of varying stiffness during consecutive crash phases has been observed. In spite of this fact, up to now, to describe such a phenomenon only one non-linear method had been introduced. The low accuracy of velocity determination using energetic methods seems to be hard to overcome. On the contrary, great accuracy enhancement and more complete physical picture of the deformation is offered by the non-linear method. Using such an approach, it was possible for the authors to decrease the pre-crash speed determination error to 6–8% [18,19], while the error in the linear method reaches the order of 30%.

2. Non-linear approach

The non-linear approach has another crucial advantage. Methods of crash data analysis (deformation coefficients' measurement) that are currently used are based on the application of energetic methods determine speed. The models currently existing rely on the assumption of a relation between Equivalent Energy Speed (EES) and the Cs ratio. The L_t parameter that represents the width of the area of deformation in a plane perpendicular to the vehicle axis is also included in each model. This kind of approach simplifies calculations to a large extent and leads to lower requirements of computing power and higher stability of solution convergence. These programs are distributed as ready-made modules which do not provide the required accuracy for the determined quantity.

The EES and Cs ratio was presumed to be the bgk coefficient. Bgk coefficient is generally constant or close to constant. The initial value of EES for Cs=0 is defined by this coefficient. The value of speed, for which, despite of the collision, there is not any deformation of the body, is represented by the bgk coefficient. This is corresponding to the ideally elastic crash. 3.05 m/s is the evaluation of the value of this coefficient.

It is assumed that an electronic module based on ranging laser will be developed in the future by the authors. The value of pre-crash speed would be instantly shown upon measuring coefficients C1-C6 using such a module. Naturally, without using a computer, the complex formulas of energetic method are not applied easily. Instead of intermediate coefficient calculations, it is suggested to compute the work W directly from coefficient Cs with the use of the function of a known algebraic form. Therefore, the precise speed Vt before the crash would be given by the vehicle mass input. The algebraic form of a function, which calculated with non-linear estimation algorithms that approximate the experimental data in the best way, were determined by the authors. What is more, its mathematical form allow a direct application, as is it relatively simple.

A couple of parameters determined by the fitting of the model to the set of existing data ale included in the function. The elaborated model may be used as a tool that describes many vehicle classes as well as their sub-types, and enables to give precise values of speed right before impact.

The authors have paid attention also to another issue – the non-centricity of deformation coefficients (C1-C6) connected with the crash test itself. The method of tensor B-spline products with probabilistic weight was applied to define the excess of this non-centricity. When it comes to the SubCompact Class, the deformation coefficients that are both symmetrical on both sides of the vehicle chassis and those not symmetrical were taken into consideration by the authors. Physical traits of the car design as well as its geometry could result in such a asymmetry. On the one hand, non-symmetrical stiffness during the deformation is shown by the engine bay equipment, but simultaneously, after the crash elements of a car body retain their original shape to some extent in a different way on the left and on the right side of the vehicle.

In spite of the fact that the range of mass in the SubCompact Class is narrow, the data structure is bound up with the vehicle brand. What is obvious, control points C1-C6 (pictures after the impact) present a seemingly chaotic broken line. The correlation between the respective values of such coefficients is analyzed in this paper by the authors.

3. Description of B-splines

Let $(x_i)_{i=1}^{I}$ be a set of nodes. First, let's define:

$$\boldsymbol{B_i^0}(\boldsymbol{x}) = \begin{cases} \mathbf{1} \ \mathbf{if} \ \boldsymbol{x} \in [\boldsymbol{x_i}, \boldsymbol{x_{i+1}}) \\ \mathbf{0} \qquad \mathbf{otherwise} \end{cases}. \tag{2}$$

They are so-called B-splines of order 0 [1,3]. From a mathematical point of view, they are characteristic functions of intervals $[x_i, x_{i+1}]$.

Recursive formula (cf. [31], p. 90), by Carl de Boor, allows us to design B-splines of order $d \geq 1$:

$$\boldsymbol{B_i^d}(\boldsymbol{x}) = \frac{x-x_i}{x_{i+d}-x_i}\boldsymbol{B_i^{d-1}}(\boldsymbol{x}) + \frac{x_{i+d+1}-x}{x_{i+d+1}-x_{i+1}}\boldsymbol{B_{i+1}^{d-1}}(\boldsymbol{x}), \tag{3}$$

B-splines have the following very practical properties: for example,

$B_i^d > 0$ in the range $[x_i, x_{i+d+1}]$ and $B_i^d = 0$ out of this range. Furthermore, the following equality takes place:

$$\sum_i \boldsymbol{B_i^d} = \mathbf{1} \ \ \mathbf{for} \ [\mathbf{x_d}, \mathbf{x_{I-d}}\ ]. \tag{4}$$

In other words, the B-splines of the d-order form the partition of one on $[x_d, x_{I-d}]$ (compare [31], page 96). Interestingly, there is a close relationship between the number of nodes, the degree of B-splines and their number:

number of B-splines = number of nodes - degree of B-spline.

For example, if we want to consider 4 splines 4, we need exactly 8 nodes.

4. Approximation of tensor B-spline products with probabilistic weights

Let a set of points be given $(x_n, y_n, z_n)_{n=1}^{N}$. Suppose that we can use the function family $(h_m)_{m=1}^{M}$ for the least-square function approximation (these are functions of two variables). In other words, our goal is to find coefficients $(a_m)_{m=1}^{M}$, which minimize the expression

$$\sum_{n=1}^{N} \boldsymbol{w_n}\left(\boldsymbol{z_n} - \sum_{m=1}^{M} \boldsymbol{a_m} \boldsymbol{h_m}(\boldsymbol{x_n}, \boldsymbol{y_n})\right)^2. \tag{5}$$

Note the entered weights $(w_n)_{n=1}^N$, which intuitively (indirectly) map the meaning of points (x_n, y_n, z_n). The lower the value of the weight, the lower the significance of (x_n, y_n, z_n).

In the case where $(a_m)_{m=1}^M$, the point is completely ignored.

We will now describe one of the options for choosing weights. This is not a complicated process.

We put

$$w_n = \frac{\text{number of points } (x_n', y_n', z_n') \text{ with } \left| z_n' - z_n \right| < \frac{\mathrm{Var}(z)}{4}}{N}. \tag{6}$$

Intuitively, if the points are close to each other, then we should consider them as "more important" than isolated points. In other words, more weight is given to clusters of points.

It can be shown that the problem of a least-square approximation can be reduced to the solution of a linear equation:

$$\begin{pmatrix} \sum_{n=1}^N w_n h_1(x_n, y_n) h_1(x_n, y_n) & \cdots & \sum_{n=1}^N w_n h_1(x_n, y_n) h_M(x_n, y_n) \\ \vdots & \ddots & \vdots \\ \sum_{n=1}^N w_n h_M(x_n, y_n) h_1(x_n, y_n) & \cdots & \sum_{n=1}^N w_n h_M(x_n, y_n) h_M(x_n, y_n) \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ \vdots \\ a_M \end{pmatrix} = \begin{pmatrix} \sum_{n=1}^N w_n z_n h_1(x_n) \\ \vdots \\ \sum_{n=1}^N w_n z_n h_M(x_n) \end{pmatrix} \tag{7}$$

In our case, the role of the function $(h_m)_{m=1}^M$ will be performed by tensor products of B-splines. Functions $(f_i)_{i=1}^5$ and $(g_j)_{j=1}^5$ are the first 5 B-splines of order 4.

Then 25 tensor products are as follows:

$$h_1 = f_1 \otimes g_1, \;\; h_2 = f_1 \otimes g_2, \;\; h_3 = f_1 \otimes g_3, \;\; h_4 = f_1 \otimes g_4, h_5 = f_1 \otimes g_5,$$

$$h_6 = f_2 \otimes g_1, \;\; h_7 = f_2 \otimes g_2, \;\; h_8 = f_2 \otimes g_3, h_9 = f_2 \otimes g_4, h_{10} = f_2 \otimes g_5,$$

$$h_{11} = f_3 \otimes g_1, h_{12} = f_3 \otimes g_2, h_{13} = f_3 \otimes g_3, h_{14} = f_3 \otimes g_4, h_{15} = f_3 \otimes g_5,$$

$$h_{16} = f_4 \otimes g_1, h_{17} = f_4 \otimes g_2, h_{18} = f_4 \otimes g_3, h_{19} = f_4 \otimes g_4, h_{20} = f_4 \otimes g_5,$$

$$h_{21} = f_5 \otimes g_1, h_{22} = f_5 \otimes g_2, h_{23} = f_5 \otimes g_3, h_{24} = f_5 \otimes g_4, h_{25} = f_5 \otimes g_5.$$

5. Results of approximation of B-spline tensor products

The data set includes 465 crash tests. The model is built on the basis of 80% of our data, then it is validated using the remaining elements of the test set (20% of data). For every single pair of variables Cs and m that is analyzed, differentiation has been made between values of speed Vr obtained from taught model (80% of data) and those obtained from validating model (20% of data) through calculation. In accordance with the assumptions, method error was represented by such differences.

First, we attach a Figure 1 showing the weights in our model.

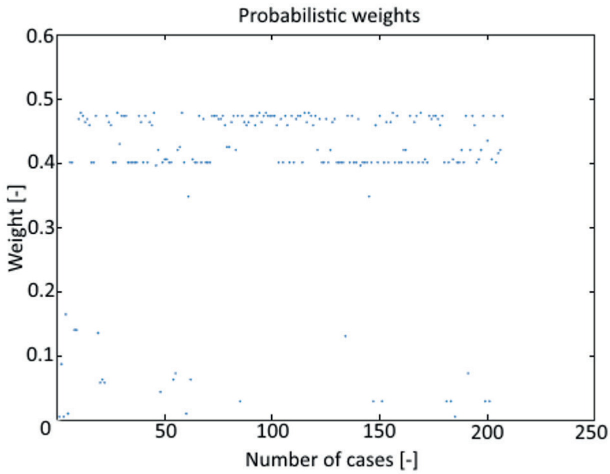

Figure 1. Probabilistic weights against number of case used in the model.

Our model has been implemented in Matlab. The program returned the following values of coefficients $(\boldsymbol{a_m})_{\boldsymbol{m=1}}^{\boldsymbol{M}}$:

$a_1 = 431.006941$; $a_2 = -712.568781$; $a_3 = 915.974211$; $a_4 = -794.550521$;

$a_5 = 538.027981$; $a_6 = -400.291994$; $a_7 = 857.781453$; $a_8 = -1113.953185$;

$a_9 = 1065.949685$; $a_{10} = -666.281811$; $a_{11} = 350.310303$; $a_{12} = -879.648878$;

$a_{13} = 1299.871487$; $a_{14} = -1203.652187$; $a_{15} = 805.272163$; $a_{16} = -191.176141$;

$a_{17} = 838.233283$; $a_{18} = -1265.718279$; $a_{19} = 1262.148553$; $a_{20} = -794.701507$;

$a_{21} = 80.677369$; $a_{22} = -690.319607$; $a_{23} = 1192.136218$; $a_{24} = -1146.131065$;

$a_{25} = 772.031756$.

The chart of the approximation of the function $\boldsymbol{V_r} = \mathbf{f}(\boldsymbol{C_s}, \boldsymbol{m})$ with B-spline tensor products for the analyzed data set was attached in the Figure 2.

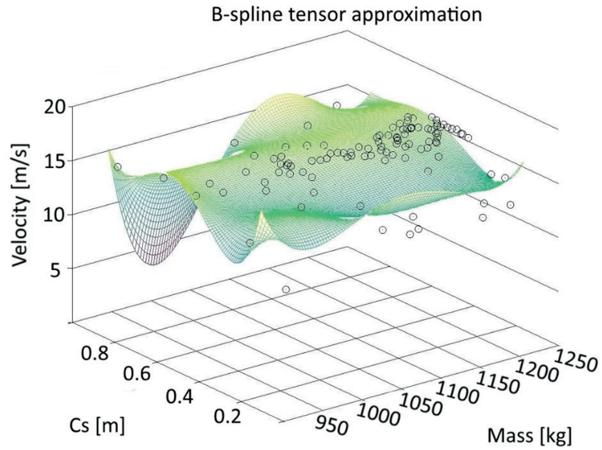

Figure 2. Approximation of the function V_r=f(C_s, m) with B-spline tensor products for the analyzed data set.

For comparison, a linear approximation is given by the formula:

$$(C_s, m) \to 7.521549 + 12.452939 \cdot m + 0.003178 \cdot C_s - (-0.004769) \cdot m \cdot C_s \;\; [-] \qquad (8)$$

looks like in the Figure 3:

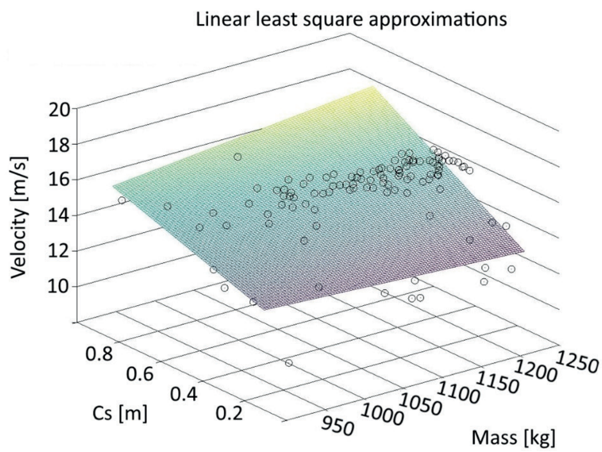

Figure 3. A linear approximation of the function V_r=f(m,C_s).

The B-splines [functions $(f_i)_{i=1}^{5}$] we used in the non-linear model are presented in the Figure 4. The B-splines [functions $(g_j)_{j=1}^{5}$] are presented in the Figure 5.

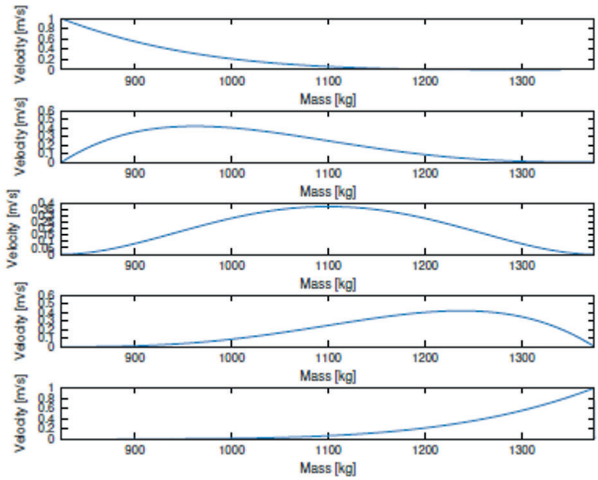

Figure 4. The B-splines [functions $(\mathrm{f_i})_{i=1}^{5} \equiv V_r = \mathrm{f}(m)$ when $C_{si} = \mathrm{const}$] used in the non-linear model.

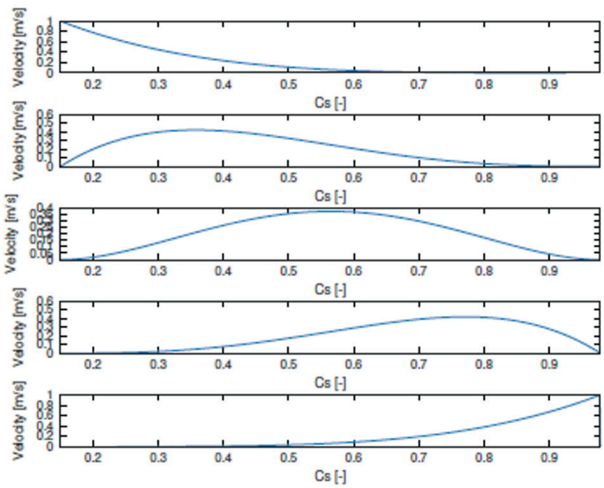

Figure 5. The B-splines [functions $(\mathrm{g_j})_{j=1}^{5} \equiv \mathrm{V_r} = \mathrm{f}(\mathrm{C_s})$ when $\mathrm{m_j} = \mathrm{const}$] used in the non-linear model.

The obtained tensor products of B-splines are shown in the Figure 6.

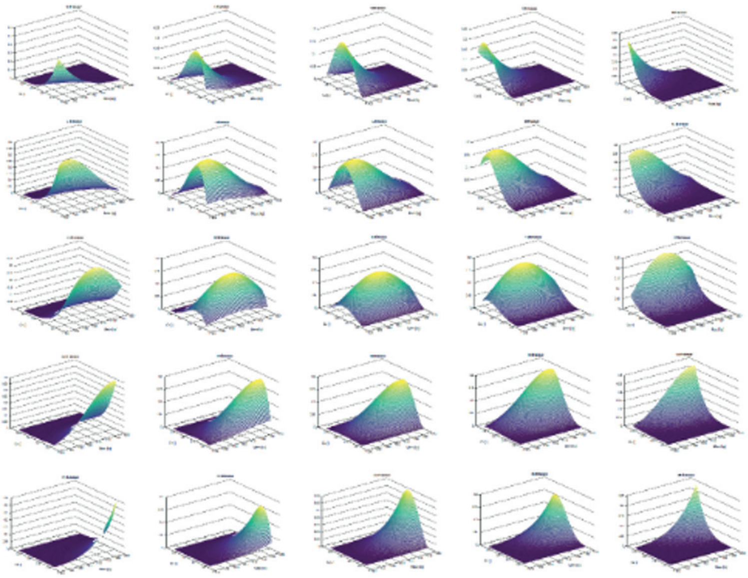

Figure 6. The obtained tensor products of B-splines used in the non-linear model.

The obtained values of the error in our method were presented in the Figure 7.

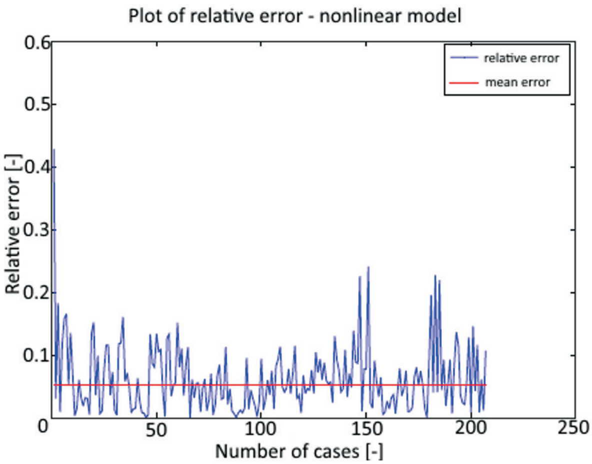

Figure 7. The obtained values of the error in our method for analyzed pairs of variables C_s and m.

The red line indicates the error of the cases given by the formula:

$$e_{weighted} = \frac{\sum_{n=1}^{N} w_n \left| \frac{B(x_n, y_n) - z_n}{z_n} \right|}{\sum_{n=1}^{N} w_n} \ [-] \qquad (9)$$

where B is a B-spline approximating the analyzed points, z_n is the initial value and the whole numerator poses as the relative error in point (x_n, y_n). The relative weighted error is 0.046459 ≈ 4.6459%. For comparison, a weighted relative error for the linear model is 0.055644 ≈ 5.5644%.

Ultimately, we choose 31 crash tests from the entire base number 465. On these selected tests, we compare the linear and nonlinear models. Below we place a Table 1, which is summarized in the following chart (Fig. 8).

Table 1. Parameters and errors for Intermediate Car Class.

Mass	C_s	V_t	Expected linear	Expected nonlinear	Linear Error	Nonlinear Error
948	0.166900	9.722222	13.010364	13.887428	0.338209	0.428421
1098	0.396000	13.138889	14.019434	13.558678	0.067018	0.031950
1202	0.605700	15.611111	15.452103	15.544785	0.010186	0.004249
1090	0.342800	13.000000	13.712743	12.939937	0.054826	0.004620
951	0.539600	16.055556	14.657937	14.755577	0.087049	0.080967
1048	0.535600	15.833333	14.768765	14.722438	0.067236	0.070162
1224	0.617800	15.694444	15.581254	15.571986	0.007212	0.007803
1166	0.539200	15.611111	14.948556	15.389389	0.042441	0.014203
1015	0.363700	13.333333	13.856985	13.848489	0.039274	0.038637
1200	0.443800	13.194444	14.344473	14.411353	0.087160	0.092229
1116	0.204200	11.111111	12.865727	13.786032	0.157915	0.240743
1213	0.443200	15.555556	14.346141	14.333243	0.077748	0.078577
1229	0.274260	11.083333	13.154800	13.248686	0.186899	0.195370
1144	0.380300	13.250000	13.920446	13.565603	0.050600	0.023819

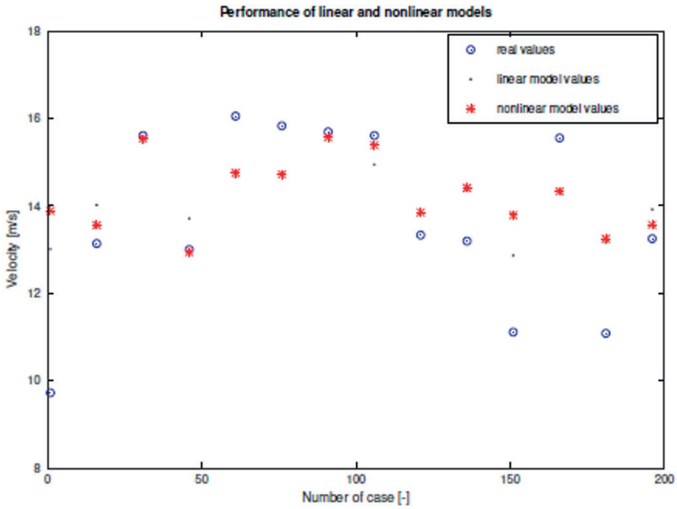

Figure 8. Performance of linear and non-linear models.

6. Conclusions

Following the method proposed by the author further, it may be able to modify the energy method approach, as well as improve the precision of vehicle velocity determination by several percent. The method would keep its practical character, enabling it to be used in a real collision, as well as it would be low cost approach that does not require as much analysis like in other cases. The author will apply for the access to NHTSA databases for modern car crash analyses, with possibly more records and wider spectrum of vehicle velocities. Author also plans on perfecting the approach in the direction of nonlinear zone model---nonlinear model with corrugation of this nonlinearity in different intervals of increasing chassis deformation. This would correspond to the attempt at capturing absolutely impractical relations of varying chassis stiffness during bending. This way the approximation of theoretical plots of exerted force and average deformation will serve for optimization of precision for vehicle velocity determination.

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